systematica investments

Example Questions 2018

1. Consider an urn which contains 1 red and 1 yellow ball at time 0. An instant before time 1, 2, 3, etc. you choose a ball at random from the urn and replace it together with a new ball of the same color. Calculate the probability $p_{n,r} \coloneqq P(at\ time\ n\ the\ urn\ contains\ r\ red\ balls)$. Extend and solve the problem to a general starting point: the urn contains r red and y yellow balls at time 0. Hint: think about the triple defining the state of the urn: (n,r,y).

2. A cone is inscribed in the sphere of radius R. Find the cone with maximum volume. Find the cone with maximum lateral surface. Compare the two solutions and comment.

3. Two points A and B are chosen independently in the unit disc. Assume D to be the distance between them. Find $E(D^2)$.

4. Consider a circle and an equilateral triangle inscribed inside the circle. Write some code to compute the following by simulation:

    a. Take two points randomly on the circumference of the circle. Connect and measure the length of the chord through the two points. What is the probability that the chord is longer than the side of the triangle?
    b. Take a random point inside a circle. This is the midpoint of only one chord. What is the probability that this chord is longer than the side of the triangle?
    c. Take two points randomly inside a circle. Consider the chord that goes through the two points. What is the probability that the chord is longer than the side the triangle?

5. When a tourist arrives in a country capital W they want to visit also two main cities: G in the north and K in the south. From survey data, when in W, there is the probability *p* of the tourist to go to K, 0 to G and 1-*p* to remain in W. When in K, *p* to go to G, 1-*p* to return to W, and 0 to continue to stay in K. Finally, when in G, 1-*p* to go to K, 0 to go to W and *p* to continue in G. Hint: think about mathematical formalism for transition problems.

    a. Is there a distribution of tourists between W, K and G that remains invariant in time?
    b. Assuming the start of the trip in W, what are the probabilities of being in W, K or G after *n* days? Please provide a closed form formula.

6. Write a function that solves a labyrinth defined by a matrix and starting in the top left corner and finishing in the bottom right corner. The input is an NxM matrix with entries either 0 or 1, where 0 means a cell where you can walk and 1 means a cell with a wall where you cannot walk. The function should return:

    a. Whether there is a path through the labyrinth
    b. The length of the shortest path through the labyrinth